Laser-Induced Thickness Stretch Motion of a Transversely Constrained Irradiated Slab

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This paper is concerned with the dynamic response of a transversely constrained, but arbitrarily thick slab due to a Holobeam model 630-Q Nd pulsed laser irradiation normal to the surface of the slab. The laser beam is assumed to be Gaussian in space and time. The laser beam energy density is adjusted to cause heating of the slab, but to avoid phase change (melting) of the slab material. The initial thickness stretch motion occurs in a time period that is very small compared to the gross motion of the thick slab with edges far away from the laser beam effective diameter. Hence, during the initial stages of the thickness stretch motion, the transversely constrained irradiated slab is considered a one-dimensional problem through the thickness with a uniform laser irradiation over the effective laser beam diameter. Solutions are obtained for the time-dependent deformation and stress fields through the thickness of the slab within the framework of classical uncoupled dynamic thermoelasticity theory and classical methods in boundary value problems. The plots confirm a wave nature of deflections that various points through the thickness are undisturbed until the wave reaches the points. Also, interesting observations are made regarding sign reversals of the deflections and stresses that are useful for the designers.

Nomenclature

Dimensional Quantities

c = specific heat
C_L = dilatational wave speed, $=\sqrt{\lambda+2G/\rho}$
C_T = shear wave speed, $=\sqrt{G/\rho}$
E = Young's modulus
h = thickness
I_m = maximum laser irradiance
k = thermal diffusivity
K = thermal conductivity
p = fundamental period of the medium
R = reflectivity of the material-laser
t = time
t_p = pulse duration of the laser
u_i, u = components of displacement vector
x_i = body force
α = coefficient of thermal expansion of the material of the solid
$\bar{\alpha}$ = absorption coefficient of the solid material
β $=(3\lambda+2G)\alpha$
θ = temperature induced in the solid due to laser irradiation on the surface
κ^2 $=C_L^2/C_T^2=(\lambda+2G)/G=2(1-\nu)/(1-2\nu)$,

$$\delta_{ij}=\begin{cases}0 & i\neq j\\ 1 & i=j\end{cases}$$

λ, G = Lame's constants
$\bar{\mu}$ = absorption depth of the solid material, $=1/\bar{\alpha}$
ρ = mass density
$\tau_{ij}, \tau_{xx}, \tau_{zz}$ = components of stress tensor
T_i^ν = traction vector on surface S with outward normal ν_i
Ω = frequency

Nondimensional Quantities

See Table 1

Subscripts and Superscripts

$(\dot{\ })$ = differentiation with respect to time
$(\ )_{,}$ = differentiation, i.e., $u_{i,j}=\partial u_i/\partial x_j$

Subscripts i, j, and k will assume integer values 1-3, respectively. Repeated subscripts imply summation over all values that the subscript may take.

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Introduction

LASERS have been utilized for such diverse purposes as the splitting of rock, machining and shaping of metals, fusion welding of bones and metals, and repair of detached retinas. They have also been considered as an unconventional line of sight weapon.

It is well known that lasers have the inherent ability to deposit focused, radiant energy on a structure or its elements. Lasers can concentrate a short-duration, high-energy flux in an extremely narrow beam. The effect of short-duration, high-energy irradiation on an opaque solid can have several forms, described (roughly) as follows (see Ref. 1, Chap. 3):

1) Complete local vaporization of the material, and the resulting creation of openings (holes). If, in addition, the structure so punctured is in a state of initial stress, there will be an additional dynamic effect due to unloading waves and wave reflections, with the possibility of stress intensification.

2) Sudden deposition of thermal energy, without a change in phase. This causes sudden thermal stresses in the structure and, because of the rapidity of the energy deposition process, there will be thermally generated stress waves.

3) It is possible for the structure to experience partial surface evaporation over the effective laser beam cross section. This results in material removal and plasma generation.

Subsequent heating of the plasma gives rise to shock wave formation, resulting in impulse transmitted to the solid.

Effects 1-3 can coexist, but they can be separated for analytical purposes. The present analytical investigation is concerned with effect (case) 2.

Laser heating of solids and the resulting motion due to sudden thermal stresses have been studied by Morland,[2] Rausch,[3] Sve and Miklowitz,[4] and others. Morland[2] describes one-dimensional stress waves generated by electromagnetic radiation using a simple temperature profile for a particular type of material in which the diffusion time is considerably greater than the wave travel time within the absorption layer. Rausch[3] investigated the effect of reducing heating time with respect to the material response time. Peak stresses were obtained as the result of this change in the time scale. Sve and Miklowitz[4] deal with the generation of stress waves in an infinite slab due to a spatially Gaussian laser beam using Hankel transform techniques.

More recently, laser heating of finite bodies and the resulting motion has been predicted using the improved plate theory.[5,6] Both of these investigations used a more appropriate laser heating model for solids proposed by Bechtel,[7] in which the laser pulse was assumed to be Gaussian in space and time. Reference 5 shows experimental evidence of a circular plate response and Ref. 6 of a rectangular plate response to laser irradiation on the surfaces of the plates. Of late, the dynamic response of a simply supported rectangular plate of arbitrary thickness to laser irradiation normal and central to the plate surface using the elasticity theory has also been achieved.[8] An initial thickness stretch motion of the plate occurs over the effective diameter of the laser beam in a time period that is very small compared to the gross motion of the plate.

The present investigation is concerned with the dynamic response associated with the initial thickness stretch motion of the plate using Bechtel's model of surface heat generation. The plate is treated as a transversely constrained slab uniformly irradiated over the effective diameter of the laser beam. The development is based on the equations of the uncoupled dynamic thermoelasticity theory, as the effects of thermoelastic damping due to the mechanical coupling term in the energy equation are negligible for both metals and the times of the several response cycles of interest here.[9] The density of laser beam energy is adjusted to cause heating of the slab, but to avoid any phase change (melting) of the slab material. Thus, we can maintain the maximum induced surface temperature within one-third of the melting temperature of the slab material, so that any viscoelastic effects can be neglected.[10] Temperature-sensitive material properties such as thermal conductivity, coefficient of thermal expansion, etc., are also considered as constants under the restrictions of the temperature rise. The temperature profile thus arrived at as the result of the above restrictions is used in the thermoelastic problem. The solutions are obtained for the time-dependent deformation and stress fields in the slab within the framework of classical thermoelasticity theory using classical methods in boundary value problems.[11,12]

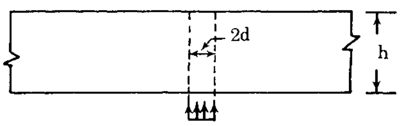

Fig. 1 Plate with laser spot.

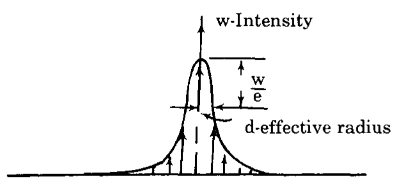

Fig. 2 Spatial distribution of laser beam.

Description of the Boundary Value Problem

A transversely constrained slab of arbitrary thickness is irradiated normal to the bottom surface of the slab as shown in Fig. 1. The spatial distribution of a Q-switched, pulsed laser beam Gaussian in space and time is shown in Fig. 2. The slab is subjected to a uniform laser intensity of radiation over the laser beam effective diameter, as shown in exaggerated form in Fig. 3.

The temperature profile $\theta(z,t)$ due to a Q-switched, pulsed laser, uniform in space and Gaussian in time is obtained from Ref. 7 after suitable modifications and is given by

$$\theta(z,t) = \frac{I_m(1-R)\sqrt{t_p}}{(\pi K\rho c)^{1/2}} \int_0^\infty \frac{e^{-[(t/t_p)-\tau]^2} e^{-(z+h/2)^2/4kt_p\tau}}{\sqrt{\tau}} \, d\tau \qquad (1)$$

Equation (1) assumes $h^2 \gg 4kt$, where h is the thickness of semi-infinite solid, k the diffusivity of the material of the solid, and t the time of interest here.

A plot of temperature induced at the bottom surface of the slab vs time is shown in Fig. 4.

Basic Equations and Statement of Boundary Value Problem in Thermoelasticity

The Navier's equation of motion of a linearly elastic, isotropic solid within the framework of small-deformation

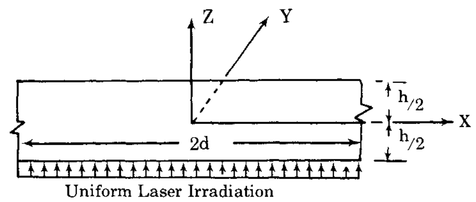

Fig. 3 Transversely constrained and uniformly irradiated slab.

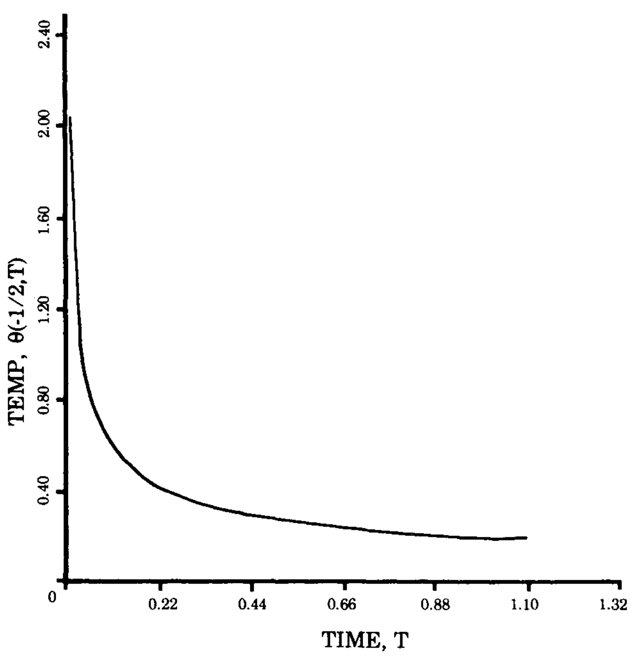

Fig. 4 Temperature induced at the bottom of the slab vs time.

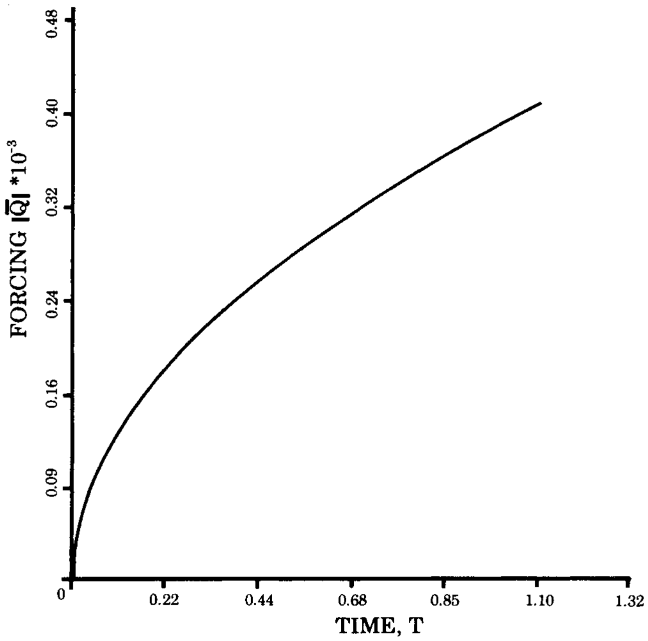

Fig. 5 Forcing function vs time for the fundamental mode.

theory is given by[9,13]

$$Gu_{i,jj} + (\lambda+G)u_{j,ji} + X_i - \beta\theta,_i = \rho\ddot{u}_i \tag{2}$$

Since we are dealing with a medium $-\infty < X < \infty$, $-\infty < Y < \infty$, and $-h/2 \leq Z \leq h/2$ (see Fig. 3) and for the displacement assumptions,

$$u_3 \equiv u(z,t) \tag{3}$$

$$u_2 = u_1 \equiv 0 \tag{4}$$

Eq. (2) reduces to

$$(\lambda+2G)\frac{\partial^2 u}{\partial z^2} - \beta\theta,_z = \rho\ddot{u} \tag{5}$$

where the body force X_i has been omitted for convenience. The associated stress boundary conditions are

$$\tau_{zz}\left(-\frac{h}{2},t\right) = f(t) = 0 \qquad \tau_{zz}\left(+\frac{h}{2},t\right) = g(t) = 0 \tag{6}$$

and we specify,

$$-\beta\theta,_z \text{ in } V \tag{7}$$

The stress tensor is given by

$$\tau_{zz} = (\lambda+2G)\frac{\partial u}{\partial z} - \beta\theta \tag{8}$$

Also, we impose the following initial conditions in V:

$$u(z,0) = u^{(0)}(z) \equiv 0 \qquad \dot{u}(z,0) = \dot{u}^{(0)}(z) \equiv 0 \tag{9}$$

General Solutions

Eigenvalue and Eigenfunction

Setting the thermal field $\theta \equiv 0$ in Eqs. (5-8) and following the standard procedure[14] for a "free-vibrations" problem, the eigenfunctions $U^{(m)}(z)$ and eigenvalues Ω_m are obtained as

$$U^{(m)}(z) = \sqrt{\frac{2}{h\rho}}\left(\cos\frac{m\pi}{2}\cos\frac{m\pi}{h}z + \sin\frac{m\pi}{2}\sin\frac{m\pi}{h}z\right) \tag{10}$$

$$\Omega_m = \frac{m\pi}{h}\sqrt{\frac{\lambda+2G}{\rho}} \tag{11}$$

where the eigenfunctions $U^{(m)}(z)$ have been uniquely determined by arbitrarily setting

$$\int_{-h/2}^{+h/2} \rho U^{(m)}(z)U^{(n)}(z)\mathrm{d}z = \delta_{mn} \tag{12}$$

and $m = 1,2\ldots$ and $n = 1,2\ldots.$

Method of Solution

The total solution of Eq. (5) is assumed to be

$$u(z,t) = u^{(s)}(z,t) + \sum_{m=1}^{\infty} U^{(m)}(z)q^{(m)}(t) \tag{13}$$

where the "quasistatic solution" $u^{(s)}(z,t)$ satisfies Eq. (5) with the inertia term set to zero and also the homogeneous stress conditions at the top and bottom surfaces of the slab.

Table 1 Nomenclature: nondimensional quantities

Nondimensional quantity	To convert to dimensional form multiply by	Physical interpretation
$\lvert\bar{Q}\rvert$	$\frac{2\beta I_m(1-R)\sqrt{t_p}\bar{\gamma}}{(\pi K\rho c)^{1/2}}\sqrt{\frac{2}{h\rho}}\frac{h^2}{C_T^2}$	Forcing
T, T_c	$h\sqrt{\rho/G}$	Time
T_{xx}	$\frac{2\beta I_m(1-R)\sqrt{t_p}\bar{\gamma}}{(\pi K\rho c)^{1/2}}$	Component of stress tensor in x direction
T_{zz}	$\frac{2\beta I_m(1-R)\sqrt{t_p}\bar{\gamma}}{(\pi K\rho c)^{1/2}}$	Component of stress tensor in z direction
U	$\frac{2\beta I_m(1-R)\sqrt{t_p}\bar{\gamma}h}{G(\pi K\rho c)^{1/2}}$	Component of displacement vector in z direction
$\bar{z}$	h	z coordinate
$\bar{\gamma} = \sqrt{kt_p/h^2}$	—	—
θ	$\frac{I_m(1-R)\sqrt{t_p}}{(\pi K\rho c)^{1/2}}$	Temperature
ν	—	Poisson's ratio
ν_i	—	Direction cosines of exterior normal to surface S

Quasistatic Solutions

The quasistatic solution as obtained following the above procedure is given by

$$(\lambda+2G)u^{(s)}(z,t)=\frac{2\beta I_m(1-R)t_p\sqrt{k}}{(\pi K\rho c)^{1/2}}\times\frac{\pi}{4}\left[\operatorname{erf}\frac{z+(h/2)}{(4kt)^{1/2}}-\operatorname{erf}\frac{h/2}{(4kt)^{1/2}}\right]\left[1+\operatorname{erf}\left(\frac{t}{t_p}\right)\right] \quad (14)$$

and the stresses are given by

$$\tau_{zz}^{(s)}(z,t)\equiv 0 \quad (15)$$

Forced Motion

Substituting Eq. (13) into Eq. (5) and making use of both the "free vibrations" and "quasistatic" solutions result in the following expression:

$$\sum_{m=1}^{\infty}(\ddot{q}^{(m)}+\Omega_m^2 q^{(m)})\rho U^{(m)}=-\rho\ddot{u}^{(s)} \quad (16)$$

Multiplying both sides of Eq. (16) by $U^{(n)}$ and integrating later through the thickness and using Eq. (12) results in

$$\ddot{q}^{(m)}+\Omega_m^2 q^{(m)}=\ddot{Q}^{(m)}(t) \quad (17)$$

where

$$Q^{(m)}(t)=-\int_{-h/2}^{+h/2}\rho u^{(s)}U^{(m)}\mathrm{d}z \quad (18)$$

Evaluation of Eq. (18) after considerable manipulation results in

$$Q^{(m)}(t)=\frac{2\beta I_m(1-R)\sqrt{t_p}\bar{\gamma}}{(\pi K\rho c)^{1/2}}\sqrt{\frac{2}{h\rho}}\frac{h^2}{C_T^2}\frac{(-1)^m}{m\pi\kappa^2}\times\int_0^{\infty}e^{-[(t/t_p)-\tau]^2}I(m,\tau)\mathrm{d}\tau \quad (19)$$

where

$$\bar{\gamma}=(kt_p)^{1/2}/h \quad (20)$$

$$I(m,\tau)=\int_0^{h\sqrt{\eta}}e^{-\zeta^2}\sin\frac{m\pi\zeta}{h\sqrt{\eta}}\mathrm{d}\zeta \quad (21)$$

$$\eta=1/4kt_p\tau \quad (22)$$

The solution of Eq. (17) for homogeneous initial conditions is written as

$$q^{(m)}(t)=Q^{(m)}(t)-\Omega_m\int_0^t Q^{(m)}(\xi)\sin\Omega_m(t-\xi)\mathrm{d}\xi \quad (23)$$

Total Solutions

Now, the total solution for the displacement is obtained by substituting Eqs. (14) and (23) into Eq. (13). Hence,

$$u(z,t)=\frac{2\beta I_m(1-R)t_p\sqrt{k}}{(\pi K\rho c)^{1/2}(\lambda+2G)}\frac{\pi}{4}\left[\operatorname{erf}\frac{z+(h/2)}{(4kt)^{1/2}}-\operatorname{erf}\frac{h/2}{(4kt)^{1/2}}\right]\left[1+\operatorname{erf}\left(\frac{t}{t_p}\right)\right]+\sum_{m=1}^{\infty}U^{(m)}(z)q^{(m)}(t) \quad (24)$$

Substituting Eq. (13) in Eq. (7) and using Eq. (15), we get

$$\tau_{zz}(z,t)=(\lambda+2G)\sum_{m=1}^{\infty}\frac{\mathrm{d}U^{(m)}}{\mathrm{d}z}q^{(m)}(t) \quad (25)$$

Also, the longitudinal stress $\tau_{xx}(z,t)$ is given by

$$\tau_{xx}(z,t)=-\frac{2\beta G\theta(z,t)}{(\lambda+2G)}+\lambda\sum_{m=1}^{\infty}\frac{\mathrm{d}U^{(m)}}{\mathrm{d}z}q^{(m)}(t) \quad (26)$$

Discussion of Results

The slab considered in this analysis is 0.254 cm thick. The laser used is Q-switched, plused, and uniform in space and Gaussian in time. The laser beam has a pulse duration of 10

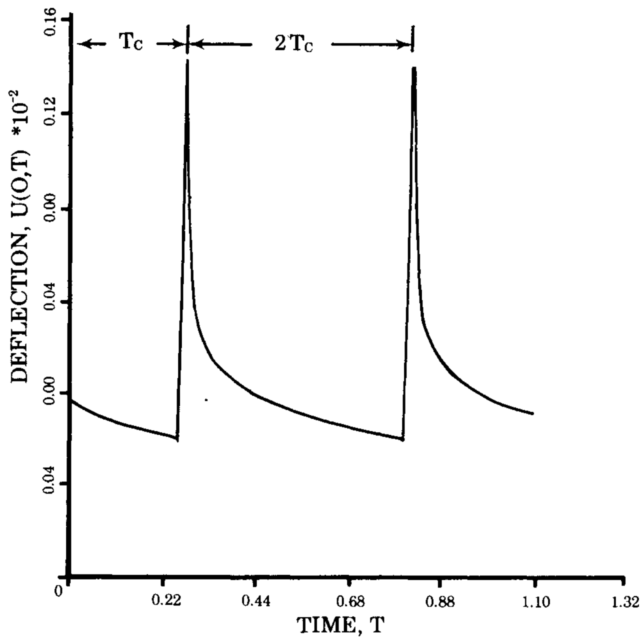

Fig. 6 Deflection at the middle surface vs time.

Table 2 Data used in the slab analysis

Particulars	Values/units
Laser parameters	
Maximum irradiance I_m	cal/s·cm^2
Pulse duration t_p	10^{-8} s
Reflectivity R	—
Material properties (steel)	
Thermal properties	
Thermal conductivity K	0.0441 cal/s·cm·°C
Thermal diffusivity k	0.0468 cm^2/s
Specific heat c	0.1195 cal/g·°C
Coefficient of thermal expansion α	cm/cm·°C
Optical properties	
Absorption depth ($\bar{\mu}=1/\bar{\alpha}$, where $\bar{\alpha}$ is the absorption coefficient)	cm
Mechcanical properties	
Mass density ρ	7.86 g/cm^3
Young's modulus E	2.1×10^6 kg/cm^2
Poisson's ratio ν	0.3
Shear modulus G	0.808×10^6 kg/cm^2
Physical quantities	
Thickness h	0.254 cm
Fundamental period, p	0.8539×10^{-6} s

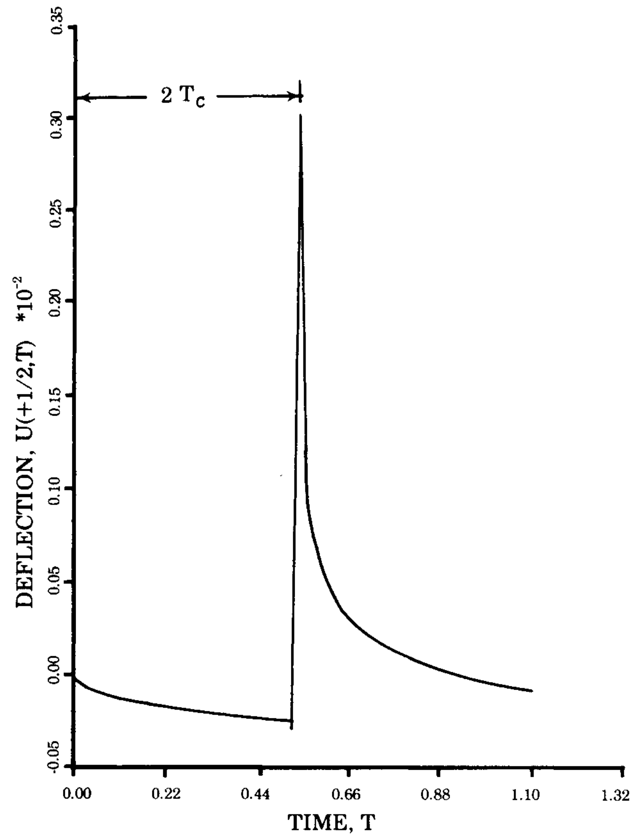

Fig. 7 Deflection at the top surface vs time.

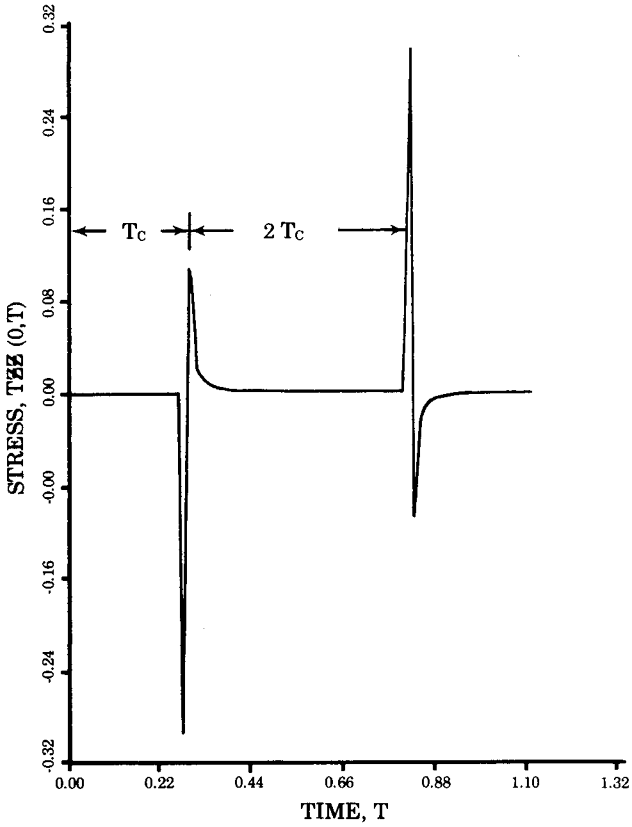

Fig. 9 Normal stress at the middle surface vs time.

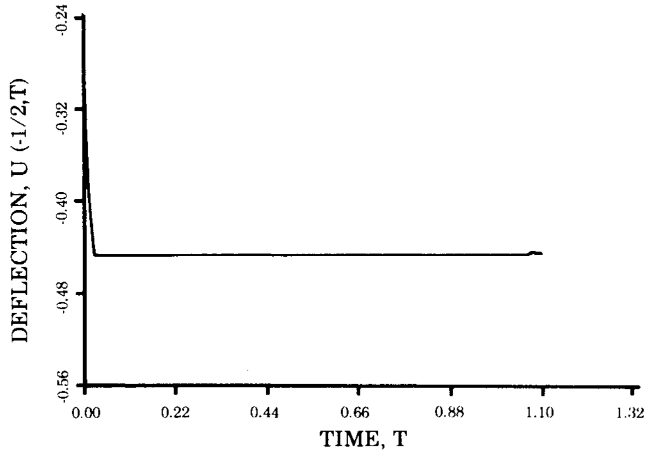

Fig. 8 Deflection at the surface of laser irradiation vs time.

ns. The mechanical and thermal properties of steel at room temperature are found in Ref. 15. Table 2 gives the complete list of data used in this analysis.

The temperature profile used in this analysis is justified for finite thickness of the medium and for the times of interest involving several cycles of response of the medium, by satisfying the inequality, $h^2 \gg 4kt$. A nondimensional plot of the temperature induced at the surface of laser irradiation is obtained from Eq. (1) and is shown in Fig. 4. The forcing function $Q^{(m)}(t)$ given by Eq. (19) is evaluated for the fundamental mode $(m=1)$ and the nondimensional plot is shown in Fig. 5. Also, Lanczos' smoothing technique[16-18] is used for faster convergence and the summation indicated in Eqs. (24-26) are carried up to 110 terms to get a reasonable amount of accuracy.

The deflection u given by Eq. (24) is evaluated at the middle, top, and bottom surfaces of the slab and is shown in Figs. 6-8, in nondimensional form. The stress τ_{zz} given by

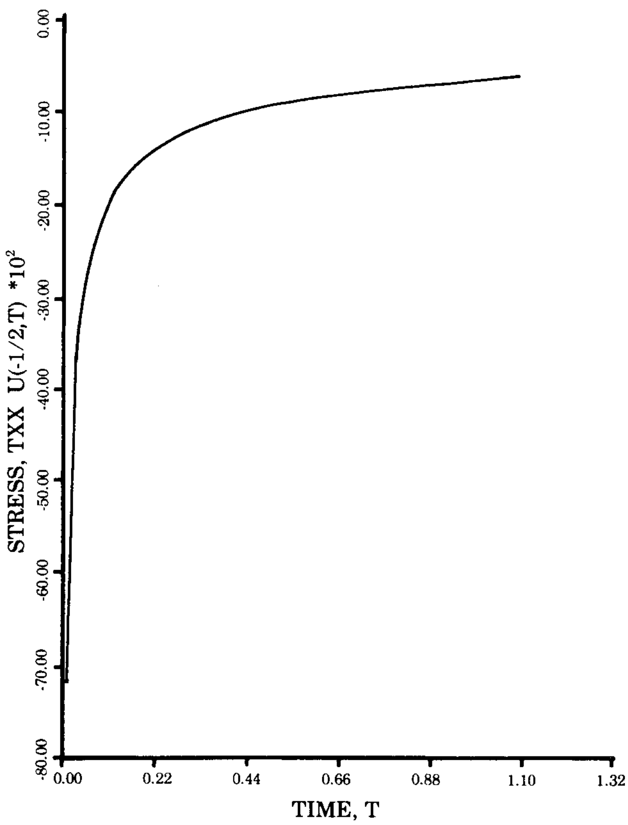

Fig. 10 Longitudinal stress at the surface of the laser irradiation vs time.

Eq. (25) is evaluated at the median surface and is shown in Fig. 9. Also, the longitudinal stress τ_{xx} given by Eq. (26) is evaluated at the surface of laser irradiation and is shown in Fig. 10.

The characteristic time T_c indicated in these figures is the amount of time taken for the dilatational stress wave to travel through half of the thickness of the slab. It is interesting to see from Figs. 6 and 9 that the slabs show no appreciable response (deflection of stress at the middle surface) until the dilatational wave reaches the middle surface from the surface of laser irradiation. Hence, the first peak occurs at T_c in these figures. The stress wave leaving the middle surface travels through the other half of the thickness of the slab, reflects at the top, and travels back toward the middle surface; hence, the second peak or response occurs after $2T_c$ from the first peak.

Similarly Fig. 7 shows no appreciable response until the dilatational wave reaches the top surface. Figure 10 shows the high compressive longitudinal stress developed in the beginning at the surface of laser irradiation and quickly decreasing later. For the design purposes, it is important to take into account the sign reversal of both the deflections and stresses evidenced in these figures.

It is worth mentioning that in this analysis a nondimensional time of $1 \cdot 1$ (more than the period) has been equally divided into 800 divisions and $q^{(m)}(t)$ given by Eq. (23) is evaluated numerically by parabolic interpolation scheme described in Refs. 5 and 19.

Acknowledgment

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